Supersymmetric Higgs Yukawa Couplings to Bottom Quarks at next-to-next-to-leading Order

David Noth[1,2] and Michael Spira[1]

[1] *Paul Scherrer Institut, CH–5232 Villigen PSI, Switzerland*
[2] *Institut für Theoretische Physik, Zürich University, CH–8057 Zürich, Switzerland*

Abstract

The effective bottom Yukawa couplings are analyzed for the minimal supersymmetric extension of the Standard Model at two-loop accuracy within SUSY–QCD. They include the resummation of the dominant corrections for large values of tgβ. In particular the two-loop SUSY-QCD corrections to the leading SUSY-QCD and top-induced SUSY-electroweak contributions are addressed. The residual theoretical uncertainties range at the per-cent level.

1 Introduction

The Standard Model (SM) predicts the existence of one scalar Higgs boson which constitutes the remainder of electroweak symmetry breaking by means of the Higgs mechanism [1]. In all experiments this particle has escaped detection so far. Due to the hierarchy problem in the context of Grand Unified Theories supersymmetric (SUSY) extensions of the SM are considered as the most attractive solutions [2]. The minimal supersymmetric extension of the SM (MSSM) requires the existence of five elementary Higgs bosons, two neutral CP-even (scalar) bosons h, H, one neutral CP-odd (pseudoscalar) boson A and two charged bosons $H^{\pm}$. At lowest order all couplings and masses of the MSSM Higgs sector are fixed by two independent input parameters, which are generally chosen as tg$\beta = v_2/v_1$, the ratio of the two vacuum expectation values $v_{1,2}$, and the pseudoscalar Higgs mass M_A. Including the one-loop and dominant two-loop corrections the upper bound on the light scalar Higgs mass is $M_h \lesssim 135$ GeV [3]. More recent first three-loop results confirm this upper bound within less than 1 GeV [4]. The couplings of the various Higgs bosons to fermions and gauge bosons depend on mixing angles α and β, which are defined by diagonalizing the neutral and charged Higgs mass matrices. They are collected in Table 1 relative to the SM Higgs couplings. For large values of tgβ the down-type Yukawa couplings are strongly enhanced, while the up-type Yukawa couplings are suppressed, unless the light (heavy) scalar Higgs mass ranges at its upper (lower) bound, where the couplings become SM-like. This feature causes the dominance of bottom-Yukawa-coupling induced processes for large values of tgβ at present and future colliders as Higgs decays into bottom quarks and Higgs bremsstrahlung off bottom quarks at hadron and e^+e^- colliders. Moreover, Higgs boson production via gluon fusion $gg \to h, H, A$ is dominated by the bottom-loop contributions for

1

Φ		g_u^Φ	g_d^Φ	g_V^Φ
SM	H	1	1	1
MSSM	h	$\cos\alpha/\sin\beta$	$-\sin\alpha/\cos\beta$	$\sin(\beta-\alpha)$
	H	$\sin\alpha/\sin\beta$	$\cos\alpha/\cos\beta$	$\cos(\beta-\alpha)$
	A	$1/\tan\beta$	$\tan\beta$	0

Table 1: MSSM Higgs couplings to SM particles relative to SM Higgs couplings.

large tgβ. Thus, the bottom Yukawa coupling determines the profile of the MSSM Higgs bosons for large tgβ to a large extent.

The negative direct searches for the MSSM Higgs bosons at LEP2 yield lower bounds of $M_{h,H} > 92.8$ GeV and $M_A > 93.4$ GeV. The range $0.7 < \text{tg}\beta < 2.0$ in the MSSM is excluded by the Higgs searches for a SUSY scale $M_{SUSY} = 1$ TeV at the LEP2 experiments [5]. Presently and in the future Higgs bosons can be searched for at the Tevatron at Fermilab [6], a proton-antiproton collider with a center-of-mass energy of 1.96 TeV, and the proton-proton collider LHC (Large Hadron Collider) with 14 TeV center-of-mass energy [7] as well as a future linear e^+e^- collider with a center-of-mass energy up to about 1 TeV [8].

The mass degeneracy of the fermions f and their superpartners $\tilde{f}$ is removed by the soft SUSY breaking terms, which induce mixing of the current eigenstates $\tilde{f}_L$ and $\tilde{f}_R$ in addition. The sfermion mass matrix in the current eigenstate basis is given by[1]

$$\mathcal{M}_{\tilde{f}}^2 = \begin{pmatrix} M_{LL}^2 & M_{LR}^2 \\ M_{RL}^2 & M_{RR}^2 \end{pmatrix} = \begin{pmatrix} M_{\tilde{f}_L}^2 + m_f^2 & m_f(A_f - \mu r_f) \\ m_f(A_f - \mu r_f) & M_{\tilde{f}_R}^2 + m_f^2 \end{pmatrix} \tag{1}$$

with the parameters $r_d = 1/r_u = \text{tg}\beta$ for down- and up-type sfermions. The mass eigenstates $\tilde{f}_{1,2}$ of the sfermions $\tilde{f}$ are related to the current eigenstates $\tilde{f}_{L,R}$ by mixing angles θ_f,

$$\begin{aligned} \tilde{f}_1 &= \tilde{f}_L \cos\theta_f + \tilde{f}_R \sin\theta_f \\ \tilde{f}_2 &= -\tilde{f}_L \sin\theta_f + \tilde{f}_R \cos\theta_f, \end{aligned} \tag{2}$$

which are proportional to the masses of the ordinary fermions. Thus mixing effects are only important for the third-generation sfermions $\tilde{t}, \tilde{b}, \tilde{\tau}$. The mixing angles acquire the form

$$\sin 2\theta_f = \frac{2m_f(A_f - \mu r_f)}{m_{\tilde{f}_1}^2 - m_{\tilde{f}_2}^2} \quad , \quad \cos 2\theta_f = \frac{M_{\tilde{f}_L}^2 - M_{\tilde{f}_R}^2}{m_{\tilde{f}_1}^2 - m_{\tilde{f}_2}^2} \tag{3}$$

and the masses of the squark mass eigenstates are given by

$$m_{\tilde{f}_{1,2}}^2 = m_f^2 + \frac{1}{2}\left[M_{\tilde{f}_L}^2 + M_{\tilde{f}_R}^2 \mp \sqrt{(M_{\tilde{f}_L}^2 - M_{\tilde{f}_R}^2)^2 + 4m_f^2(A_f - \mu r_f)^2} \right]. \tag{4}$$

The topic of this paper is the calculation of the NNLO SUSY-QCD and top-induced SUSY-electroweak corrections to the effective bottom Yukawa couplings. These results will affect all processes to which the bottom Yukawa couplings contribute, i.e. in particular the Higgs decay widths and Higgs radiation off bottom quarks at hadron colliders which constitutes the dominant Higgs boson production channel for large tgβ at the Tevatron and the LHC [9].

---

[1]For simplicity, the D-terms have been absorbed in the sfermion mass parameters $M_{\tilde{f}_{L/R}}^2$.

2 Effective Bottom Yukawa Couplings

The leading parts of the SUSY–QCD and SUSY–electroweak corrections to bottom Yukawa coupling induced processes can be absorbed in effective bottom Yukawa couplings. These contributions correspond to the limit of heavy supersymmetric particle masses compared to the typical energy scales of the particular process. The accuracy of this heavy mass approximation has been investigated for neutral MSSM Higgs decays into bottom quarks $h/H/A \to b\bar{b}$ [10], charged Higgs decays to top and bottom quarks $H^\pm \to tb$ [11] and Higgs radiation off bottom quarks at e^+e^- colliders [12] and hadron colliders [13, 14] by comparing it to the full NLO results. For large values of $\mathrm{tg}\beta$ the approximation turns out to agree with the NLO results to better than one per cent.

2.1 Effective Lagrangian

The leading corrections can be obtained from the effective Lagrangian [10, 11]

$$
\begin{aligned}
\mathcal{L}_{eff} &= -\lambda_b \overline{b_R} \left[\phi_1^0 + \frac{\Delta_b}{\mathrm{tg}\beta} \phi_2^{0*} \right] b_L + h.c. \\
&= -m_b \bar{b} \left[1 + i\gamma_5 \frac{G^0}{v} \right] b - \frac{m_b/v}{1 + \Delta_b} \bar{b} \left[g_b^h \left(1 - \frac{\Delta_b}{\mathrm{tg}\alpha\,\mathrm{tg}\beta} \right) h \right. \\
&\qquad \left. + g_b^H \left(1 + \Delta_b \frac{\mathrm{tg}\alpha}{\mathrm{tg}\beta} \right) H - g_b^A \left(1 - \frac{\Delta_b}{\mathrm{tg}^2\beta} \right) i\gamma_5 A \right] b
\end{aligned}
\tag{5}
$$

with the one-loop expressions ($C_F = 4/3$) [15]

$$
\begin{aligned}
\Delta_b^{(1)} &= \Delta_b^{QCD\,(1)} + \Delta_b^{elw\,(1)} \\
\Delta_b^{QCD\,(1)} &= \frac{C_F}{2} \frac{\alpha_s(\mu_R)}{\pi} m_{\tilde{g}}\, \mu\, \mathrm{tg}\beta\, I(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2, m_{\tilde{g}}^2) \\
\Delta_b^{elw\,(1)} &= \frac{\lambda_t^2(\mu_R)}{(4\pi)^2} A_t\, \mu\, \mathrm{tg}\beta\, I(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2, \mu^2)
\end{aligned}
\tag{6}
$$

The generic function I is defined as

$$
I(a, b, c) = \frac{ab \log \dfrac{a}{b} + bc \log \dfrac{b}{c} + ca \log \dfrac{c}{a}}{(a-b)(b-c)(a-c)}
\tag{7}
$$

The fields ϕ_1^0 and ϕ_2^0 denote the neutral components of the Higgs doublets coupling to down- and up-type quarks, respectively. They are related to the mass eigenstates h, H, A by

$$
\begin{aligned}
\phi_1^0 &= \frac{1}{\sqrt{2}} \left[v_1 + H \cos\alpha - h \sin\alpha + iA \sin\beta - iG^0 \cos\beta \right] \\
\phi_2^0 &= \frac{1}{\sqrt{2}} \left[v_2 + H \sin\alpha + h \cos\alpha + iA \cos\beta + iG^0 \sin\beta \right]
\end{aligned}
\tag{8}
$$

3

The two vacuum expectation values are related to the Fermi constant G_F, $v^2 = v_1^2 + v_2^2 = 1/\sqrt{2}G_F$. The would-be Goldstone field G^0 is absorbed by the Z boson and generates its longitudinal component. The top Yukawa coupling λ_t is related to the top mass by $m_t = \lambda_t v_2/\sqrt{2}$ at lowest order. The corrections Δ_b induce a modification of the relation between the bottom quark mass m_b and the bottom Yukawa coupling λ_b,

$$m_b = \frac{\lambda_b v_1}{\sqrt{2}} \left[1 + \Delta_b \right] \tag{9}$$

The effective Lagrangian of Eq. (5) can be parametrized as

$$\mathcal{L}_{eff} = -\frac{m_b}{v} \, \bar{b} \left[\, \tilde{g}_b^h \, h + \tilde{g}_b^H \, H - \tilde{g}_b^A \, i\gamma_5 \, A \, \right] b \tag{10}$$

with the effective (resummed) couplings

$$
\begin{aligned}
\tilde{g}_b^h &= \frac{g_b^h}{1 + \Delta_b} \left[1 - \frac{\Delta_b}{\mathrm{tg}\alpha\,\mathrm{tg}\beta} \right] \\
\tilde{g}_b^H &= \frac{g_b^H}{1 + \Delta_b} \left[1 + \Delta_b \frac{\mathrm{tg}\alpha}{\mathrm{tg}\beta} \right] \\
\tilde{g}_b^A &= \frac{g_b^A}{1 + \Delta_b} \left[1 - \frac{\Delta_b}{\mathrm{tg}\beta^2} \right]
\end{aligned}
\tag{11}
$$

Although the SUSY corrections Δ_b are loop suppressed, they turn out to be significant for large values of $\mathrm{tg}\beta$ and moderate or large μ values. In these cases they constitute the dominant supersymmetric radiative corrections to the bottom Yukawa couplings. It should be noted that the effective Lagrangian in Eq. (5) has been derived by integrating out the heavy SUSY particles so that it is not restricted to large values of $\mathrm{tg}\beta$ only. In order to improve the perturbative result it has been shown that the Lagrangian of Eq.(5) resums all terms of $\mathcal{O}\left[(\alpha_s\,\mu\,\mathrm{tg}\beta)^n\right]$ and $\mathcal{O}\left[(\lambda_t^2\,A_t\,\mathrm{tg}\beta)^n\right]$ [11]. The additional resummation of terms proportional to $\alpha_s A_b$ [10] will be neglected in this paper.

2.2 Low Energy Theorems

The determination of higher-order corrections to the effective bottom Yukawa couplings would usually require the calculation of the corresponding three-point functions. This calculation can be reduced to the evaluation of self-energy diagrams by the use of low energy theorems [16]. The basic idea is that any matrix element with an external Higgs boson can be related to the analogous matrix element without the external Higgs particle in the limit of vanishing Higgs momentum by the simple replacements $v_1 \to \sqrt{2}\phi_1^0$ and $v_2 \to \sqrt{2}\phi_2^{0*}$ in the latter. Thus we only need to compute the corresponding pieces of the bottom quark self-energy. The leading pieces Δ_b emerge from the scalar part $\Sigma_S(m_b)$ of the self-energy[2] giving rise to the following

---

[2]The fermionic self-energy can be decomposed into a scalar, vectorial and axial-vectorial part according to $\Sigma(p) = \Sigma_S(p) + \not{p}\,\Sigma_V(p) + \not{p}\gamma_5\,\Sigma_A(p)$.
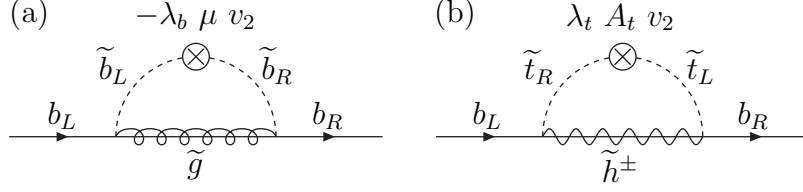
Figure 1: *One-loop diagrams of (a) the SUSY-QCD and (b) the top-induced SUSY-electroweak contributions to the bottom self-energy with the off-diagonal mass insertions corresponding to the corrections Δ_b of the bottom Yukawa couplings. The virtual particles involve bottom quarks b and squarks $\tilde{b}$, top squarks $\tilde{t}$, gluinos $\tilde{g}$ and charged Higgsinos $\tilde{h}^{\pm}$.*

relation between the pole mass m_b of the bottom quark and the bottom Yukawa coupling λ_b[3],

$$m_b = \frac{\lambda_b}{\sqrt{2}} v_1 + \Sigma_S(m_b) \tag{12}$$

where the leading terms of the self-energy $\Sigma_S(m_b)$ for large values of $\text{tg}\beta$ are given by

$$\begin{aligned} \Sigma_S(m_b) &= \frac{\lambda_b}{\sqrt{2}} v_1 \, \Delta_b \\ \Delta_b &= \Delta_b^{QCD} + \Delta_b^{elw} \end{aligned} \tag{13}$$

The terms Δ_b^{QCD} and Δ_b^{elw} can be derived from off-diagonal mass insertions of the type $-\lambda_b \mu v_2$ in the virtual sbottom propagators and of the type $\lambda_t A_t v_2$ in the virtual stop propagators, as is illustrated in Figure 1 at one-loop level. The result of these diagrams is given by the finite expressions of Eq.(6) after rotation of the fields in the current eigenstate basis to the mass eigenstates. These expressions are not renormalized since there is no tree-level coupling of bottom quarks to the Higgs field ϕ_2^{0*}. By means of power-counting arguments it can be proven that Eq. (5) includes and resums all leading terms of $\mathcal{O}\left[(\alpha_s \, \mu \, \text{tg}\beta)^n\right]$ and $\mathcal{O}\left[(\lambda_t^2 \, A_t \, \text{tg}\beta)^n\right]$ [10, 11].

3 NNLO Corrections

The calculation of the NNLO corrections to the effective bottom Yukawa couplings requires the determination of the leading NNLO corrections to the bottom self-energy. The full NNLO results of the bottom quark self-energy have been presented in Refs. [17, 18, 19] before. Our results for the self-energy at NNLO can be extracted from their calculation by keeping only the terms leading in $\text{tg}\beta$ in principle. The renormalization, however, has to be adjusted to our scheme in order to compare the final results. The results of Ref. [17] are only shown for equal SUSY-breaking squark masses $M_{\tilde{q}_L}^2 = M_{\tilde{q}_R}^2$ for the full relation between the pole quark mass and the $\overline{DR}$ quark mass. However, since the self-energy is a symmetric function of both

---

[3]The bottom Yukawa coupling is not renormalized at $\mathcal{O}(\alpha_s \mu \text{tg}\beta)$ and $\mathcal{O}(\lambda_t^2 A_t \text{tg}\beta)$ in the effective Lagrangian, but receives only non-leading contributions in $\text{tg}\beta$ [10], which do not contribute to the effective Lagrangian.

sbottom masses $m_{\tilde{b}_1}$ and $m_{\tilde{b}_2}$ their results for the *linear a_b* terms can be compared to our results in the equal SUSY mass case. Moreover, our calculation of the bottom self-energy has been performed for the leading term for small bottom momenta so that we can compare directly with the results of Ref. [19] which displays results in the same limit.

3.1 Two-Loop Diagrams

We have computed the supersymmetric QCD corrections to the effective bottom Yukawa couplings of Eq. (11), i.e. the SUSY–QCD corrected effective Lagrangian of Eq.(5) at NNLO. The two-loop calculation split up into the $\mathcal{O}(\alpha_s^2 \mu \mathrm{tg}\beta)$ corrections to $\Delta_b^{QCD(1)}$ and the $\mathcal{O}(\alpha_s \lambda_t^2 A_t \mathrm{tg}\beta)$ corrections to $\Delta_b^{elw(1)}$. The relevant two-loop diagrams contributing to the bottom self-energy are shown in Figure 2. For the determination of the NNLO corrections single scalar mass insertions in each bottom- and top-squark propagator analogous to the one-loop level have to be included and added in all possible ways. This means that e.g. the first diagram leads to two contributions, one with the mass insertion in the left sbottom propagator and the other with the mass insertion in the right sbottom propagator as depicted in Fig. 3. We have performed the whole calculation in dimensional regularization. The bottom quark momentum and its mass have been put to zero while keeping the bottom Yukawa coupling λ_b finite in the mass insertions and the charged Higgsino couplings. All supersymmetric particles as well as the top quark have been treated with full mass dependence[4]. Since the two-loop contributions are computed in the limit of vanishing external momenta the calculation requires the determination of the leading two-loop vacuum integrals in the heavy mass expansion. These integrals can be expressed in terms of the one-loop one-point integral $A_0(m)$ [20]

$$A_0(m) = \bar{\mu}^{4-n} \int \frac{d^n k}{(2\pi)^n} \frac{1}{k^2 - m^2} \tag{14}$$

where $\bar{\mu}$ denotes the 't Hooft mass, and the two-loop master integral $T_{134}(m_1, m_3, m_4)$ [21]

$$T_{134}(m_1, m_3, m_4) = \bar{\mu}^{2(4-n)} \int \frac{d^n k}{(2\pi)^n} \frac{d^n q}{(2\pi)^n} \frac{1}{(k^2 - m_1^2)[(k-q)^2 - m_3^2](q^2 - m_4^2)} \tag{15}$$

where we work in $n = 4 - 2\epsilon$ dimensions. Integrals with higher powers of the corresponding propagators can be reduced to the basic integrals A_0 and T_{134} by means of integration-by-parts methods [22]. In this way all singularities have been separated from the finite contributions analytically as poles in the parameter ϵ.

3.2 Renormalization

Contrary to the finite one-loop results, the two-loop corrections $\Delta_b^{(2)}$ are UV-divergent. A finite and $\bar{\mu}$-independent result is obtained after renormalization of the masses and couplings

---

[4]We have neglected the tiny contributions of $\mathcal{O}(m_t^2(A_t - \mu/\mathrm{tg}\beta)/M_{SUSY}^3)$ of the off-diagonal stop propagators in the gluino self-energy of the last two-loop diagram (for $q = t, \tilde{q} = \tilde{t}$) of Fig. 2a and in the gluino mass counter term $\delta m_{\tilde{g}}$ in Eq. (19).
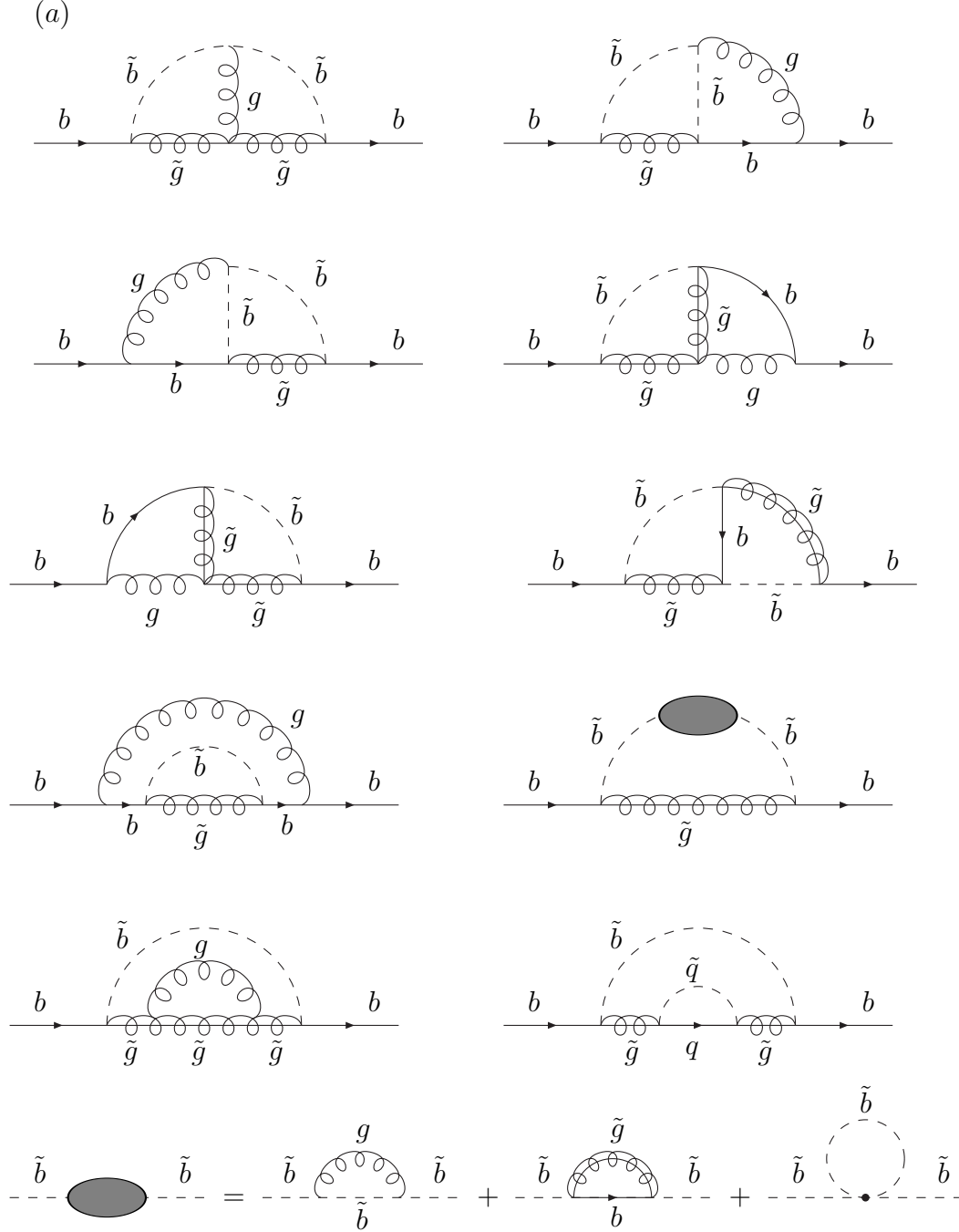
Figure 2: *Two-loop diagrams of (a) the SUSY–QCD and (b) the top-induced mixed SUSY–QCD/electroweak contributions to the bottom self-energy involving bottom quarks b, bottom $\tilde{b}$ and top $\tilde{t}$ squarks, gluons g, gluinos $\tilde{g}$ and charged Higgsinos $\tilde{h}^{\pm}$. The squark-quark contributions to the gluino propagator have to be summed over all quark/squark flavors $q/\tilde{q}$ including both directions of the flavor flow due to the Majorana nature of the gluino.*
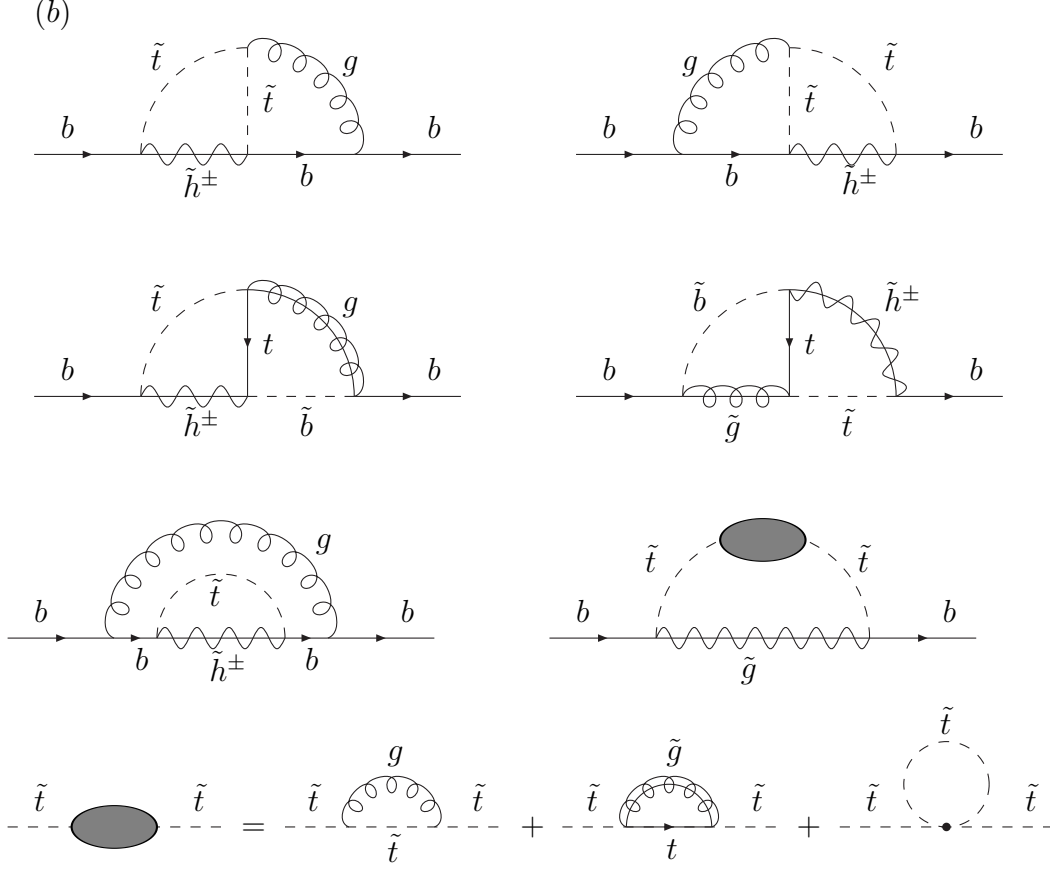
(b)

Figure 2: *Cont'd.*

contributing to the one-loop result. The heavy masses $m_{\tilde{b}_i}, m_{\tilde{t}_i}, m_{\tilde{g}}$ appearing in the propagators have been renormalized on-shell. The trilinear coupling A_t has been treated in the on-shell scheme, too, where its counter term is derived from the counter terms of the stop mixing angle and masses, and the top Yukawa coupling[5],

$$\delta A_t = \frac{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}{2m_t}\left[2\cos 2\theta_t \delta\theta_t - \sin 2\theta_t \frac{\delta m_t}{m_t}\right] + \frac{\sin 2\theta_t}{2m_t}\left[\delta m_{\tilde{t}_1}^2 - \delta m_{\tilde{t}_2}^2\right] \tag{16}$$

where δm_t denotes the on-shell counter term of the top mass,

$$\frac{\delta m_t}{m_t} = iC_F g_s^2 \left\{\frac{A_0(m_t)}{m_t^2} + 2B_0(m_t^2; 0, m_t) + B_1(m_t^2; m_{\tilde{g}}, m_{\tilde{t}_1}) + B_1(m_t^2; m_{\tilde{g}}, m_{\tilde{t}_2})\right\} \tag{17}$$

---

[5]Note that in the desired order the mixing angle counter term does not depend on the momentum of the off-diagonal self-energy contribution, since the gluino contribution does not contribute to our perturbative order, $\delta\theta_t = iC_F g_s^2 \sin 2\theta_t \cos 2\theta_t [A_0(m_{\tilde{t}_2}) - A_0(m_{\tilde{t}_1})]/(m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2)$.
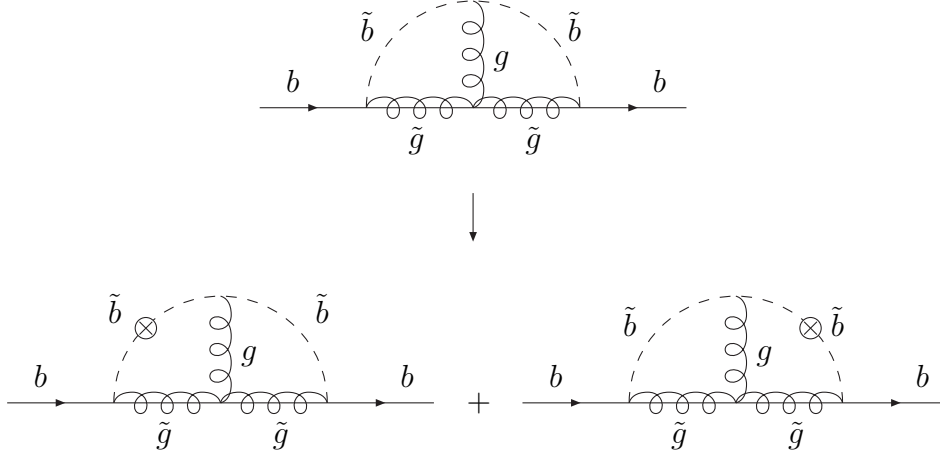
8

Figure 3: *All possible mass insertions into the first diagram of Fig. 2 contributing to the leading terms of Δ_b at NNLO.*

The strong coupling α_s and the top Yukawa coupling λ_t have been defined in the $\overline{\text{MS}}$ scheme with 5 active flavors, i.e. the top quark and the supersymmetric particles have been decoupled from the scale dependence of the strong coupling $\alpha_s(\mu_R)$. Care has to be taken to include only the desired order, i.e. $\mathcal{O}(\alpha_s^2 \mu \text{tg}\beta / M_{SUSY})$ for Δ_b^{QCD} and $\mathcal{O}(\alpha_s \lambda_t^2 A_t \text{tg}\beta / M_{SUSY})$ for Δ_b^{elw}. In this order the trilinear coupling A_t only receives a finite renormalization at NLO.

The complete set of counter terms is given by

$$
\begin{aligned}
m_{\tilde{q}_i}^{0\,2} &= m_{\tilde{q}_i}^2 + \delta m_{\tilde{q}_i}^2 \\
m_{\tilde{g}}^{0} &= m_{\tilde{g}} + \delta m_{\tilde{g}} \\
\lambda_t^{0} &= \lambda_t(\mu_R) + \delta\lambda_t \\
\alpha_s^{0} &= \alpha_s(\mu_R) + \delta\alpha_s \\
A_t^{0} &= A_t + \delta A_t
\end{aligned}
\tag{18}
$$

with the following explicit expressions ($C_A = 3$)

$$\delta m_{\tilde{t}_i}^2 = -iC_F g_s^2 \left\{ 2A_0(m_{\tilde{t}_i}) - 2A_0(m_{\tilde{g}}) - 2A_0(m_t) - 4m_{\tilde{t}_i}^2 B_0(m_{\tilde{t}_i}^2; 0, m_{\tilde{t}_i}) \right.$$
$$\left. +2(m_{\tilde{t}_i}^2 - m_{\tilde{g}}^2 - m_t^2) B_0(m_{\tilde{t}_i}^2; m_{\tilde{g}}, m_t) \right\}$$

$$\delta m_{\tilde{b}_i}^2 = -iC_F g_s^2 \left\{ 2A_0(m_{\tilde{b}_i}) - 2A_0(m_{\tilde{g}}) - 4m_{\tilde{b}_i}^2 B_0(m_{\tilde{b}_i}^2; 0, m_{\tilde{b}_i}) \right.$$
$$\left. +2(m_{\tilde{b}_i}^2 - m_{\tilde{g}}^2) B_0(m_{\tilde{b}_i}^2; m_{\tilde{g}}, 0) \right\}$$

$$\frac{\delta m_{\tilde{g}}}{m_{\tilde{g}}} = ig_s^2 \left\{ C_A \left[2B_0(m_{\tilde{g}}^2; 0, m_{\tilde{g}}) - 2B_1(m_{\tilde{g}}^2; 0, m_{\tilde{g}}) - \frac{i}{(4\pi)^2} \right] \right.$$
$$-\sum_q \left[B_0(m_{\tilde{g}}^2; m_{\tilde{q}_1}, m_q) + B_0(m_{\tilde{g}}^2; m_{\tilde{q}_2}, m_q) \right.$$
$$\left. \left. +B_1(m_{\tilde{g}}^2; m_{\tilde{q}_1}, m_q) + B_1(m_{\tilde{g}}^2; m_{\tilde{q}_2}, m_q) \right] \right\}$$

$$\frac{\delta \lambda_t}{\lambda_t} = C_F \frac{\alpha_s}{\pi} \Gamma(1+\epsilon)(4\pi)^\epsilon \frac{3}{4} \left\{ -\frac{1}{\epsilon} + \log \frac{\mu_R^2}{\bar{\mu}^2} \right\} + iC_F g_s^2 \left\{ B_1(m_t^2; m_{\tilde{g}}, m_{\tilde{t}_1}) + B_1(m_t^2; m_{\tilde{g}}, m_{\tilde{t}_2}) \right\}$$

$$\frac{\delta \alpha_s}{\alpha_s} = \frac{\alpha_s}{\pi} \Gamma(1+\epsilon)(4\pi)^\epsilon \left\{ \left(-\frac{1}{\epsilon} + \log \frac{\mu_R^2}{\bar{\mu}^2} \right) \beta_0 + \frac{C_A}{6} \log \frac{\mu_R^2}{m_{\tilde{g}}^2} + \frac{1}{6} \log \frac{\mu_R^2}{m_t^2} \right.$$
$$\left. + \sum_{\tilde{q}_i} \frac{1}{24} \log \frac{\mu_R^2}{m_{\tilde{q}_i}^2} \right\}$$

$$\frac{\delta A_t}{A_t} = -iC_F g_s^2 \left\{ \frac{A_0(m_t)}{m_t^2} + 2B_0(m_t^2; 0, m_t) + B_1(m_t^2; m_{\tilde{g}}, m_{\tilde{t}_1}) + B_1(m_t^2; m_{\tilde{g}}, m_{\tilde{t}_2}) \right.$$
$$-4\frac{m_{\tilde{t}_1}^2 B_0(m_{\tilde{t}_1}^2; 0, m_{\tilde{t}_1}) - m_{\tilde{t}_2}^2 B_0(m_{\tilde{t}_2}^2; 0, m_{\tilde{t}_2})}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}$$
$$\left. +2\frac{(m_{\tilde{t}_1}^2 - m_{\tilde{g}}^2 - m_t^2)B_0(m_{\tilde{t}_1}^2; m_{\tilde{g}}, m_t) - (m_{\tilde{t}_2}^2 - m_{\tilde{g}}^2 - m_t^2)B_0(m_{\tilde{t}_2}^2; m_{\tilde{g}}, m_t)}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} \right\} \qquad (19)$$

where the lowest order beta function is given by

$$\beta_0 = \frac{3C_A - N_F - 1}{4} \qquad (20)$$

with $N_F = 5$ light flavors ($m_q = 0$) and summing over $N_F + 1 = 6$ quark/squark flavors in the expressions above. The masses of all quarks except the top have been put to zero. In addition to the one-loop scalar integral A_0 of Eq.(14) the residual scalar one-loop integrals are defined as [20]

$$B_0(p^2; m_1, m_2) = \bar{\mu}^{4-n} \int \frac{d^n k}{(2\pi)^n} \frac{1}{(k^2 - m_1^2)[(k+p)^2 - m_2^2]}$$
$$B_1(p^2; m_1, m_2) = \frac{A_0(m_1) - A_0(m_2) - (p^2 + m_1^2 - m_2^2)B_0(p^2; m_1, m_2)}{2p^2} \qquad (21)$$

10

A few comments are in order. Since we calculate the $\mathcal{O}(\alpha_s^2 \mu \mathrm{tg}\beta)$ and $\mathcal{O}(\alpha_s \lambda_t^2 A_t \mathrm{tg}\beta)$ corrections we only included contributions to these orders in our counter terms. This plays a role for the counter terms of the gluino mass, the stop masses, the top Yukawa coupling λ_t and the trilinear coupling A_t. In the gluino mass counter term we have neglected contributions of the order $m_t/m_{\tilde{g}}$, in the counter terms of the stop masses and the top Yukawa coupling λ_t we deleted terms of $\mathcal{O}(m_t A_t)$, and finally in the counter term of the trilinear coupling A_t we only included contributions linear in A_t. Moreover, we have put the bottom mass m_b to zero everywhere apart from the Yukawa coupling λ_b in the mass insertions and the Higgsino couplings. This leads to the simplified counter terms listed above [23].

The counter term for the strong coupling constant contains the logarithmic finite terms required for the decoupling of the top quark and the supersymmetric particles from its scale dependence, i.e. the coupling α_s runs with only five light flavors,

$$\mu_R^2 \frac{\partial \alpha_s}{\partial \mu_R^2} = -\beta_0^L \frac{\alpha_s^2}{\pi} + \mathcal{O}(\alpha_s^3) \tag{22}$$

with ($N_F = 5$)

$$\beta_0^L = \frac{11 C_A - 2 N_F}{12} \tag{23}$$

Anomalous Counter Terms. A complication occurs in supersymmetric theories if dimensional regularization is employed, since in $n \neq 4$ dimensions the gluons acquire $n-2$ degrees of freedom, while the gluinos still possess 2 degrees of freedom. Thus a mismatch between the gluons and their supersymmetric partners is introduced at $\mathcal{O}(\epsilon)$ which violates the supersymmetric relations between related couplings and masses. In order to restore supersymmetry at the renormalized level finite anomalous counter terms have to be introduced which, however, are uniquely fixed by the corresponding supersymmetric identities [24].

In our calculation this mismatch of degrees of freedom between supersymmetric partners induces finite differences between the Yukawa coupling $\hat{g}_s$ of quarks, squarks and gluinos and the corresponding gauge coupling g_s,

$$\hat{g}_s = g_s \Big[1 + \frac{\alpha_s}{2\pi} \Big(\frac{C_A}{3} - \frac{C_F}{4} \Big) \Big] \tag{24}$$

and between the quark Yukawa couplings (generically denoted by λ_{Hqq}), the corresponding Higgs couplings to squarks (generically $\lambda_{H\tilde{q}\tilde{q}}$) and the Higgsino Yukawa couplings to quarks and squarks (generically $\lambda_{\tilde{H}q\tilde{q}}$),

$$\lambda_{Hqq} = \lambda_{H\tilde{q}\tilde{q}} \Big[1 + \frac{C_F}{4} \frac{\alpha_s}{\pi} \Big] = \lambda_{\tilde{H}q\tilde{q}} \Big[1 + \frac{3}{8} C_F \frac{\alpha_s}{\pi} \Big] \tag{25}$$

In our calculation we have expressed all couplings in terms of the Standard Model $\overline{MS}$ Yukawa coupling λ_{Hqq}. The anomalous counter terms for the $H\tilde{q}\tilde{q}$ vertex has already been included in the counter term δA_t given above. The left-over anomalous counter terms are those for the strong coupling $\hat{g}_s$ arising in the SUSY–QCD corrections $\Delta_b^{QCD(1)}$ at the one-loop level,

11

the bottom Yukawa coupling λ_b of the mass insertions of $\Delta_b^{QCD(1)}$, which corresponds to the generic coupling $\lambda_{H\tilde{q}\tilde{q}}$, the bottom Yukawa coupling λ_b corresponding to $\lambda_{\tilde{H}q\tilde{q}}$ generically in the SUSY-electroweak corrections $\Delta_b^{elw(1)}$ and the top Yukawa coupling factor λ_t^2 of $\Delta_b^{elw(1)}$, which corresponds to the generic product $\lambda_{H\tilde{q}\tilde{q}}\lambda_{\tilde{H}q\tilde{q}}$. This results in the following total anomalous counter terms [23]

$$
\begin{aligned}
\delta\Delta_{b;anom}^{QCD} &= \left(\frac{C_A}{3} - \frac{C_F}{4} - \frac{C_F}{4}\right)\frac{\alpha_s}{\pi}\Delta_b^{QCD(1)} = \left(\frac{C_A}{3} - \frac{C_F}{2}\right)\frac{\alpha_s}{\pi}\Delta_b^{QCD(1)} \\
\delta\Delta_{b;anom}^{elw} &= \left(-\frac{C_F}{4} - \frac{3}{8}C_F - \frac{3}{8}C_F\right)\frac{\alpha_s}{\pi}\Delta_b^{elw(1)} = -C_F\frac{\alpha_s}{\pi}\Delta_b^{elw(1)}
\end{aligned}
\tag{26}
$$

These counter terms have been added to the final results after including the counter terms of Eq.(19) to obtain consistent final results.

3.3 Resummation

According to the power-counting arguments of Ref. [10] the leading NLO contributions of $\mathcal{O}(\mu\mathrm{tg}\beta)$ can only emerge from *single* mass insertions as depicted in Fig. 1, i.e. NNLO terms of $\mathcal{O}(\mu^2\mathrm{tg}^2\beta)$ are absent, since a second mass insertion as required by the second power in $\mu\mathrm{tg}\beta$ is suppressed by an additional power of m_b/M_{SUSY}. In complete analogy the absence of $\mathcal{O}(A_t^2\mathrm{tg}^2\beta)$ in the SUSY-electroweak part Δ_b^{elw} can be proven. Thus the leading two-loop corrections are of $\mathcal{O}(\alpha_s^2\mu\mathrm{tg}\beta)$ and $\mathcal{O}(\alpha_s\lambda_t^2 A_t\mathrm{tg}\beta)$ respectively, arising from *single* mass insertions in the two-loop bottom self-energy diagrams contributing to $\Sigma_S(m_b)$ in Eq. (12). These are indeed all contributions taken into account in this work. At higher orders terms of $\mathcal{O}(\alpha_s\mu^2\mathrm{tg}^2\beta)$ and $\mathcal{O}(\alpha_s A_t^2\mathrm{tg}^2\beta)$ are absent so that our results of $\mathcal{O}(\alpha_s^2\mu\mathrm{tg}\beta)$ and $\mathcal{O}(\alpha_s\lambda_t^2 A_t\mathrm{tg}\beta)$ are automatically resummed if included in Δ_b according to Eq. (5) with

$$
\begin{aligned}
\Delta_b &= \Delta_b^{QCD} + \Delta_b^{elw} \\
\Delta_b^{QCD} &= \Delta_b^{QCD\,(1)} + \Delta_b^{QCD\,(2)} \\
\Delta_b^{elw} &= \Delta_b^{elw\,(1)} + \Delta_b^{elw\,(2)}
\end{aligned}
\tag{27}
$$

where $\Delta_b^{QCD\,(2)}$ and $\Delta_b^{elw\,(2)}$ denote our novel NNLO corrections. These expressions thus resum all terms of $\mathcal{O}(\alpha_s^n\mu^n\mathrm{tg}^n\beta)$, $\mathcal{O}(\alpha_s^{n+1}\mu^n\mathrm{tg}^n\beta)$, $\mathcal{O}(\lambda_t^{2n}A_t^n\mathrm{tg}^n\beta)$ and $\mathcal{O}(\alpha_s\lambda_t^{2n}A_t^n\mathrm{tg}^n\beta)$.

3.4 Limits

The final results for the two-loop corrections $\Delta_b^{QCD\,(2)}$ and $\Delta_b^{elw\,(2)}$ are too lengthy to be displayed here. However, they can be given explicitly in certain limits:

a) $m_t^2 \ll m_{\tilde{g}}^2 = m_{\tilde{b}_i}^2 = \mu^2 = m_{\tilde{t}_i}^2 \equiv M^2$:[6]

$$\begin{aligned}
\Delta_b^{QCD\,(1)} &= \frac{C_F}{4} \frac{\alpha_s(\mu_R)}{\pi} \mathrm{tg}\beta \\
\Delta_b^{QCD\,(2)} &= \frac{\alpha_s}{\pi} \left\{ \frac{C_A}{3} + C_F + \frac{N_F + 1}{4} + \frac{1}{6} \log \frac{M^2}{m_t^2} + \beta_0^L \log \frac{\mu_R^2}{M^2} \right\} \Delta_b^{QCD\,(1)} \\
\Delta_b^{elw\,(1)} &= \frac{1}{2} \frac{\lambda_t^2(\mu_R)}{(4\pi)^2} \frac{A_t \mathrm{tg}\beta}{M} \\
\Delta_b^{elw\,(2)} &= C_F \frac{\alpha_s}{\pi} \left\{ \frac{7}{4} + \frac{3}{2} \log \frac{\mu_R^2}{m_t M} \right\} \Delta_b^{elw\,(1)}
\end{aligned} \tag{28}$$

where terms of $\mathcal{O}(m_t^2/M^2)$ have been neglected. The logarithmic top mass term of $\Delta_b^{QCD\,(2)}$ can be absorbed in the running strong coupling by evolving it with 6 active flavors. However, since the top mass is not much lighter than the supersymmetric masses this logarithm does not become that large that this change of scheme is required. The last logarithm compensates the scale dependence of the strong coupling $\alpha_s(\mu_R)$ in the one-loop expression $\Delta_b^{QCD\,(1)}$. We have verified that our results for $\Delta_b^{QCD\,(1,2)}$ can be obtained from the X_b terms of Eq. (4.8) in Ref. [19] if the scheme transformations and decoupling relations of the strong coupling α_s and the trilinear coupling A_b are taken into account properly. Moreover, we have found agreement with the a_q terms of Eq. (61) in the first paper of Ref. [17] after appropriate scheme transformations of $\alpha_s, m_{\tilde{g}}, m_{\tilde{b}_{1/2}}$ and the bottom mass and the necessary subtraction of the product of the vectorial part of the one-loop self-energy $\Sigma_V(m_b)$ and the scalar part $\Sigma_S(m_b)$ which contributes to the relation between the pole quark mass and the $\overline{DR}$ mass in addition to the pure self-energy at the two-loop level.

The final expressions for $\Delta_b^{elw\,(2)}$ exhibits a logarithmic term of the common supersymmetric mass M which can be absorbed in the low-energy Yukawa couplings of the charged Higgsinos to stops and bottom quarks which have to be defined with properly decoupled supersymmetric contributions. These low-energy Yukawa couplings have to be evaluated at scales of the order of the top quark mass. This, however, is only necessary if the mass splitting between the top quark and the supersymmetric particles becomes large, giving rise to logarithms of their mass ratios. This does not occur in realistic scenarios so that we did not perform this low-energy decoupling.

---

[6]It should be noted that degenerate squark masses require $A_b = \mu \mathrm{tg}\beta$ and $A_t = \mu/\mathrm{tg}\beta$. Moreover, due to the different D-term contributions to the $SU(2)$-relation between the left-handed stop and sbottom mass terms the equality of the stop and sbottom masses cannot be realized. However, we display the full result in this limit as a good approximation for small mass splittings.

b) $m_t^2, m_{\tilde{b}_i}^2, \mu^2, m_{\tilde{t}_i}^2 \ll m_{\tilde{g}}^2$:

$$
\begin{aligned}
\Delta_b^{QCD(1)} &= \frac{C_F}{2} \frac{\alpha_s(\mu_R)}{\pi} \frac{\mu \mathrm{tg}\beta}{m_{\tilde{g}}} \left\{ \log \frac{m_{\tilde{g}}^2}{m_{\tilde{b}_2}^2} - \frac{m_{\tilde{b}_1}^2}{m_{\tilde{b}_2}^2 - m_{\tilde{b}_1}^2} \log \frac{m_{\tilde{b}_2}^2}{m_{\tilde{b}_1}^2} \right\} \\
\Delta_b^{QCD(2)} &= \frac{\alpha_s}{\pi} \left\{ \frac{4}{3} C_A + C_F \left[\log \frac{m_{\tilde{g}}^2}{m_{\tilde{b}_2}^2} - \frac{m_{\tilde{b}_1}^2}{m_{\tilde{b}_2}^2 - m_{\tilde{b}_1}^2} \log \frac{m_{\tilde{b}_2}^2}{m_{\tilde{b}_1}^2} + \frac{5}{2} \right] - \frac{N_F + 1}{2} \right. \\
&\quad \left. + \frac{1}{6} \log \frac{m_{\tilde{g}}^2}{m_t^2} + \frac{1}{24} \sum_{\tilde{q}} \log \frac{m_{\tilde{g}}^2}{m_{\tilde{q}}^2} + \beta_0^L \log \frac{\mu_R^2}{m_{\tilde{g}}^2} \right\} \Delta_b^{QCD(1)} \\
\Delta_b^{elw(2)} &= C_F \frac{\alpha_s}{\pi} \left\{ \frac{23}{8} + \frac{3}{2} \log \frac{\mu_R^2}{m_t m_{\tilde{g}}} \right\} \Delta_b^{elw(1)}
\end{aligned}
\tag{29}
$$

The next non-leading term in this limit is of $\mathcal{O}(\log^{-1} m_{\tilde{g}}^2 / m_{\tilde{b}_i}^2)$ for the SUSY–QCD part $\Delta_b^{QCD(2)}$ and of $\mathcal{O}(\max\{\mu^2, m_{\tilde{t}_i}^2\}/m_{\tilde{g}}^2)$ for the SUSY–electroweak part $\Delta_b^{elw(2)}$. The expression of $\Delta_b^{elw(1)}$ is not altered from Eq. (6) in this limit so that we do not display it again. The logarithms of the top and squark masses in $\Delta_b^{QCD(2)}$ can be absorbed in the running QCD coupling α_s if the latter includes the top quark and all squarks in its β function. Since the real mass splittings are not so large that these logarithms become sizeable this procedure is not necessary. Again we have verified that our results for $\Delta_b^{QCD(1,2)}$ in this limit can be derived from the X_b terms of Eq. (4.10) in Ref. [19] by taking into account the decoupling properties and scheme transformations of α_s and A_b consistently and identifying all squark masses. As for the previous limit the logarithm of the gluino mass in $\Delta_b^{elw(2)}$ can be absorbed in the low-energy Yukawa couplings of the Higgsinos to stops and bottom quarks by decoupling the gluino contributions consistently. Since these logarithms do not become large in realistic cases we did not perform this decoupling but kept the full supersymmetric relations to the related Yukawa and gauge couplings.

4 Results

The results obtained in this work have been inserted in the program HDECAY [25] which calculates the MSSM Higgs masses and couplings according to Ref. [26] as well as all partial decay widths and branching ratios including the relevant higher-order corrections [9]. For large values of $\mathrm{tg}\beta$ the decays of the neutral Higgs bosons are dominated by the decays into $b\bar{b}$ and $\tau^+\tau^-$. Their branching ratios had been studied with the one-loop expressions of the correction Δ_b of Eq. (6) in Ref. [10]. It has been demonstrated that the scale dependence of the strong coupling $\alpha_s(\mu_R)$ of Eq. (6) implies a theoretical uncertainty up to the 10% level for scenarios of large SUSY-QCD corrections to the bottom Yukawa couplings such as, e.g., the 'small α_{eff}' scenario [27].

4.1 Higgs Decays into Bottom Quark Pairs

The partial decay widths of the neutral Higgs bosons $\Phi = h, H, A$ into bottom quark pairs, including QCD and SUSY–QCD corrections, can be cast into the form [10]

$$\Gamma[\Phi \to b\bar{b}] = \frac{3G_F M_\Phi}{4\sqrt{2}\pi} \overline{m}_b^2(M_\Phi) \left[\Delta_{\text{QCD}} + \Delta_t^\Phi\right] \tilde{g}_b^\Phi \left[\tilde{g}_b^\Phi + \Delta_{SQCD}^{rem}\right] \tag{30}$$

where regular quark mass effects are neglected. The large logarithmic part of the QCD corrections has been absorbed in the running $\overline{\text{MS}}$ bottom quark mass $\overline{m}_b(M_\Phi)$ at the scale of the corresponding Higgs mass M_Φ. The QCD corrections Δ_{QCD} and the top quark induced contributions Δ_t^Φ read as [28]

$$\begin{aligned}
\Delta_{\text{QCD}} &= 1 + 5.67 \frac{\alpha_s(M_\Phi)}{\pi} + (35.94 - 1.36 N_F) \left(\frac{\alpha_s(M_\Phi)}{\pi}\right)^2 \\
&\quad + (164.14 - 25.77 N_F + 0.259 N_F^2) \left(\frac{\alpha_s(M_\Phi)}{\pi}\right)^3 \\
\Delta_t^{h/H} &= \frac{g_t^{h/H}}{g_b^{h/H}} \left(\frac{\alpha_s(M_{h/H})}{\pi}\right)^2 \left[1.57 - \frac{2}{3}\log\frac{M_{h/H}^2}{M_t^2} + \frac{1}{9}\log^2\frac{\overline{m}_b^2(M_{h/H})}{M_{h/H}^2}\right] \\
\Delta_t^A &= \frac{g_t^A}{g_b^A} \left(\frac{\alpha_s(M_A)}{\pi}\right)^2 \left[3.83 - \log\frac{M_A^2}{M_t^2} + \frac{1}{6}\log^2\frac{\overline{m}_b^2(M_A)}{M_A^2}\right]
\end{aligned} \tag{31}$$

where $N_F = 5$ active flavors are taken into account. In the intermediate and large Higgs mass regimes the QCD corrections reduce the $b\bar{b}$ decay widths by about 50% due to the large logarithmic contributions.

The main part of the SUSY–QCD corrections [29] has been absorbed in the effective bottom Yukawa couplings $\tilde{g}_b^\phi$ of Eq. (11). The remainder Δ_{SQCD}^{rem} is small in phenomenologically relevant scenarios for large values of tgβ [10]. It should be noted that this remains true even for very light SUSY masses in the range of $\mathcal{O}(100$ GeV). This can be understood by deriving the asymptotic expression of Δ_{SQCD}^{rem} for large values of the Higgs masses compared to the SUSY masses (we neglect the bottom mass here),

$$\Delta_{SQCD}^{rem} \to C_F \frac{\alpha_s(\mu_R)}{\pi} m_{\tilde{g}} (A_b - \eta_\phi \mu \text{tg}\beta) I(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2, m_{\tilde{g}}^2) \tag{32}$$

with the coefficients

$$\eta_H = \frac{\text{tg}\alpha}{\text{tg}\beta} \qquad \eta_A = -\frac{1}{\text{tg}^2\beta}$$

for the heavy scalar H and the pseudoscalar A. Thus the large Higgs mass limit approaches a finite expression of $\mathcal{O}(A_b/\mu\text{tg}\beta)$ relative to the leading Δ_b terms for large values of tgβ. This explains the validity of the Δ_b approximation even for small SUSY masses compared to the Higgs masses. In the following we include the mixed top Yukawa coupling induced SUSY–QCD/electroweak corrections in the couplings $\tilde{g}_b^\phi$, too.

4.2 Numerical Results

The numerical analysis of the neutral Higgs boson decays into bottom quark pairs is performed for two MSSM benchmark scenarios [27] as representative cases[7]:

$$\text{small } \alpha_{eff}: \quad \text{tg}\beta = 30, \quad M_{\tilde{Q}} = 800 \text{ GeV}, \quad M_{\tilde{g}} = 1000 \text{ GeV},$$
$$M_2 = 500 \text{ GeV}, \quad A_b = A_t = -1.133 \text{ TeV}, \quad \mu = 2 \text{ TeV}$$

$$\text{gluophobic}: \quad \text{tg}\beta = 30, \quad M_{\tilde{Q}} = 350 \text{ GeV}, \quad M_{\tilde{g}} = 500 \text{ GeV},$$
$$M_2 = 300 \text{ GeV}, \quad A_b = A_t = -760 \text{ GeV}, \quad \mu = 300 \text{ GeV} \tag{33}$$

We use the RG-improved two-loop expressions for the Higgs masses and couplings of Ref. [26]. Thus the leading one- and two-loop corrections have been included in the effective mixing angle α. The top pole mass has been taken as $m_t = 172.6$ GeV, while the bottom quark pole mass has been chosen to be $m_b = 4.60$ GeV, which corresponds to a $\overline{\text{MS}}$ mass $\overline{m}_b(\overline{m}_b) = 4.26$ GeV. The strong coupling constant has been normalized to $\alpha_s(M_Z) = 0.118$. We have used the top and bottom pole masses in the mass matrices of the stop and sbottom states as implemented in HDECAY.

In the following we will not address the parametric uncertainties of the corrections to the bottom Yukawa couplings and branching ratios, which will be significantly reduced after the LHC and a linear e^+e^- collider with energies in the range up to about 1 TeV [8]. However, we will concentrate mainly on the renormalization scale dependence of our final results as a first estimate of the residual theoretical uncertainties after including our novel NNLO corrections to the bottom Yukawa couplings[8]. In Fig. 4 the scale dependence of the Δ_b^{QCD} and Δ_b^{elw} terms is shown for the small α_{eff} scenario and in Fig. 5 for the gluophobic scenario. The Δ_b^{QCD} correction amounts to about 90% in the small α_{eff} scenario partially compensated by a negative Δ_b^{elw} contribution of $\mathcal{O}(10\%)$. By comparing the dashed one-loop bands with the full two-loop curves a significant reduction of the residual scale dependence can be observed leading to a final scale uncertainty in the per-cent range at NNLO. An analogous reduction of the scale dependence can be observed in the gluophobic scenario, where the Δ_b^{QCD} terms amount to about 35% and are significantly reduced by about 20% due to the electroweak Δ_b^{elw} contribution. In general, however, the sizes and signs of these contributions strongly depend on the MSSM scenario. Moreover, broad maxima/minima develop at about 1/3 of the central scales determined by the average of the corresponding SUSY-masses, i.e. $\mu_0 = (m_{\tilde{b}_1} + m_{\tilde{b}_2} + m_{\tilde{g}})/3$ for Δ_b^{QCD} and $\mu_0 = (m_{\tilde{t}_1} + m_{\tilde{t}_2} + \mu)/3$ for Δ_b^{elw} at NNLO in contrast to the monotonic scale dependence of the one-loop expressions. For the central scale choices the two-loop corrections

---

[7]Note that in the small α_{eff} scenario we increased the gluino mass by a factor of two relative to Ref. [27] in order to enhance the size of Δ_b. This value for the gluino mass has also been used in Ref. [23] so that Eq. (5) of that work has to be corrected. A general analysis of the impact of the Δ_b terms in the MSSM is beyond the scope of our work so that we restrict our numerical analysis to these two scenarios.

[8]The scheme and scale choices of the stop, sbottom and gluino masses as well as the trilinear coupling A_t modify the results by a few per cent at NLO already so that their impact ranges below the per-cent level at NNLO. These effects are neglected in our analysis, since the dominant uncertainties originate from the scale choices of the strong coupling constant α_s and the top Yukawa coupling λ_t.

to Δ_b^{QCD} amount up to $\mathcal{O}(10\%)$, while they are in the per-cent range for Δ_b^{elw} [23]. Since the corrections Δ_b enter the effective Yukawa couplings as in Eq. (11) they have an immediate impact on all processes induced by these bottom Yukawa couplings.

As a particular example we show the partial decay widths of the neutral MSSM Higgs bosons into $b\bar{b}$ pairs in Figs. 6 and 7 in the small α_{eff} and the gluophobic scenario, respectively. The two-loop corrections to Δ_b lead to negative corrections to the partial decay widths for the central scale choices of $\mathcal{O}(10\%)$. The bands at one-loop order (dashed blue curves) and two-loop order (full red curves) are defined by varying the renormalization scales of Δ_b^{QCD} and Δ_b^{elw} independently between 1/2 and 2 times the corresponding central scales μ_0. A significant reduction of the dashed one-loop bands of $\mathcal{O}(10\%)$ to the full two-loop bands at the per-cent level can be inferred from these results. The small kinks in the partial decay width $\Gamma(H \to b\bar{b})$ in the gluophobic scenario originate from the $\tilde{b}_1 \bar{\tilde{b}}_1$ and the $\tilde{b}_2 \bar{\tilde{b}}_2$ thresholds in Δ_{SQCD}^{rem} of Eq. (30). The small kink in the partial decay width $\Gamma(A \to b\bar{b})$ on the other hand can be traced back to the $\tilde{b}_1 \bar{\tilde{b}}_2$ and $\tilde{b}_2 \bar{\tilde{b}}_1$ thresholds. The light scalar Higgs boson decay width $\Gamma(h \to b\bar{b})$ exhibits a strong dependence on the pseudoscalar mass M_A for $M_A \gtrsim 120$ GeV, since it slowly approaches the SM-limit for large M_A. The drop of $\Gamma(h \to b\bar{b})$ for $M_A \sim 145$ GeV in Fig. 6 is due to the vanishing of the effective mixing angle α in the MSSM Higgs sector.

The scale uncertainties in the partial decay widths translate into theoretical uncertainties in the corresponding branching ratios of the neutral MSSM Higgs bosons. These errors cancel to a large extent in the dominant branching ratio $BR(\Phi \to b\bar{b})$, but induce sizable uncertainties of the non-leading branching ratios at one-loop order. The branching ratios for the two dominant decay modes into $b\bar{b}$ and $\tau^+\tau^-$ pairs are depicted in Figs. 8 and 9 for the small α_{eff} and the gluophobic scenario, respectively. The uncertainties of the branching ratios reduce from $\mathcal{O}(10\%)$ at one-loop order to the per-cent level at two-loop order. The per-cent accuracy now matches the expected experimental accuracies at a future linear e^+e^- collider. The thresholds visible in the branching ratios of the heavy scalar Higgs boson in Fig. 9 are due to the kinematical opening of the $\tilde{t}_1 \bar{\tilde{t}}_1, \tilde{b}_1 \bar{\tilde{b}}_1$ and $\tilde{b}_2 \bar{\tilde{b}}_2$ decay modes in consecutive order with rising Higgs mass. The thresholds of the pseudoscalar branching ratios are due to the decay modes into $\tilde{\chi}_1^0 \tilde{\chi}_2^0, \tilde{\chi}_1^+ \tilde{\chi}_1^-, \tilde{t}_1 \tilde{t}_2$ and $\tilde{b}_1 \tilde{b}_2$.

Since we have determined the effective resummed bottom Yukawa couplings at two-loop order the results will also affect all other processes which are significantly induced by bottom Yukawa couplings, as e.g. MSSM Higgs radiation off bottom quarks at e^+e^- colliders [12, 30] and hadron colliders [31]. The two-loop corrections can easily be included in the corresponding numerical programs. However, these analyses are beyond the scope of this work.

5 Conclusions

In this work we have determined the NNLO corrections to the effective bottom quark Yukawa couplings within the MSSM for large values of $\text{tg}\beta$, since the theoretical uncertainties are sizable at NLO in these regimes. The leading parts of the SUSY-QCD and top-induced SUSY-electroweak corrections originate from factorizable contributions due to virtual squark, gluino and Higgsino exchange, which can be absorbed in effective Yukawa couplings in a universal
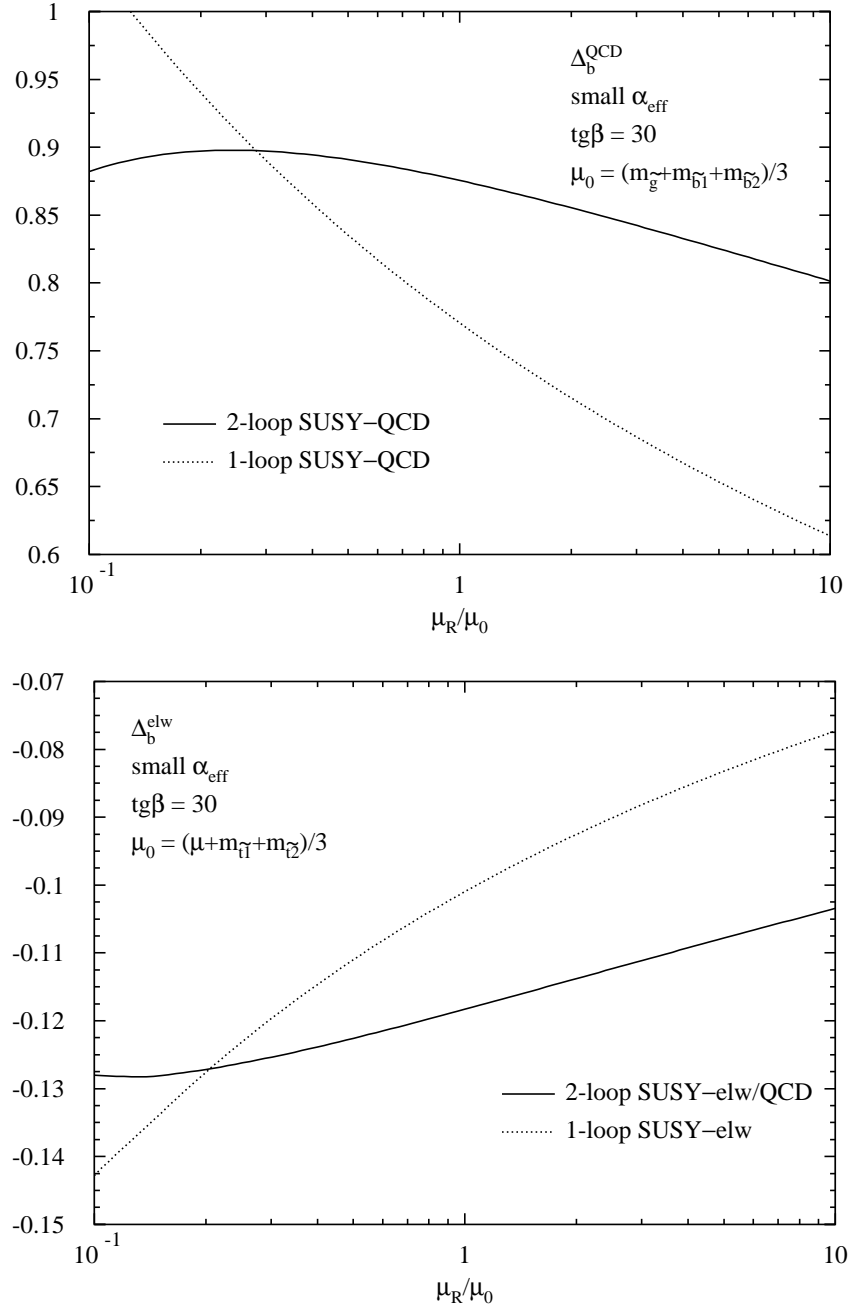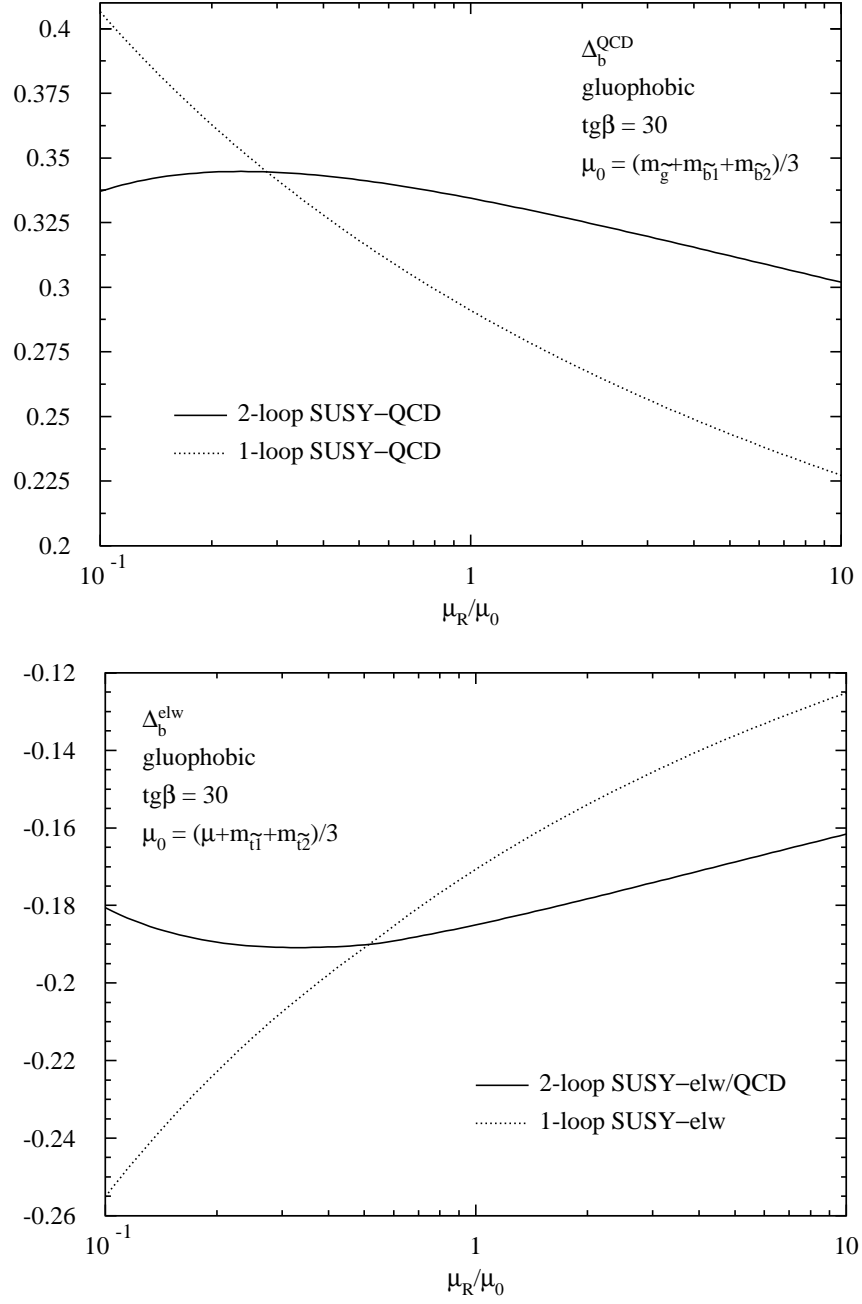
Figure 4: *Scale dependence of the SUSY–QCD correction* Δ_b^{QCD} *and the SUSY-electroweak correction* Δ_b^{elw} *at one-loop and two-loop order in the small* α_{eff} *scenario.*

Figure 5: *Scale dependence of the SUSY–QCD correction* Δ_b^{QCD} *and the SUSY-electroweak correction* Δ_b^{elw} *at one-loop and two-loop order in the gluophobic scenario.*
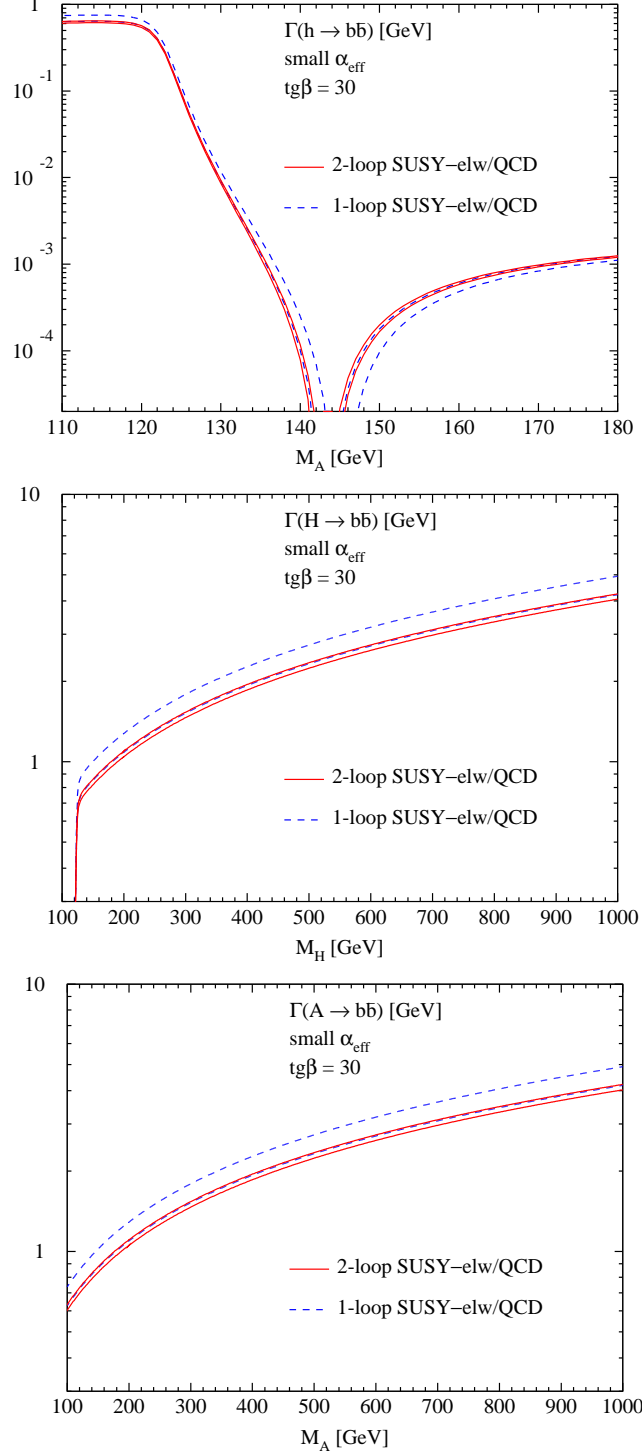
Figure 6: *Partial decay widths of the light scalar h, the heavy scalar H and the pseudoscalar A Higgs bosons to $b\bar{b}$ in the small α_{eff} scenario. The dashed blue bands indicate the scale dependence at one-loop order and the full red bands at two-loop order by varying the renormalization scales of Δ_b^{QCD} and Δ_b^{elw} independently between 1/2 and 2 times the central scale given by the corresponding average of the SUSY-particle masses.*
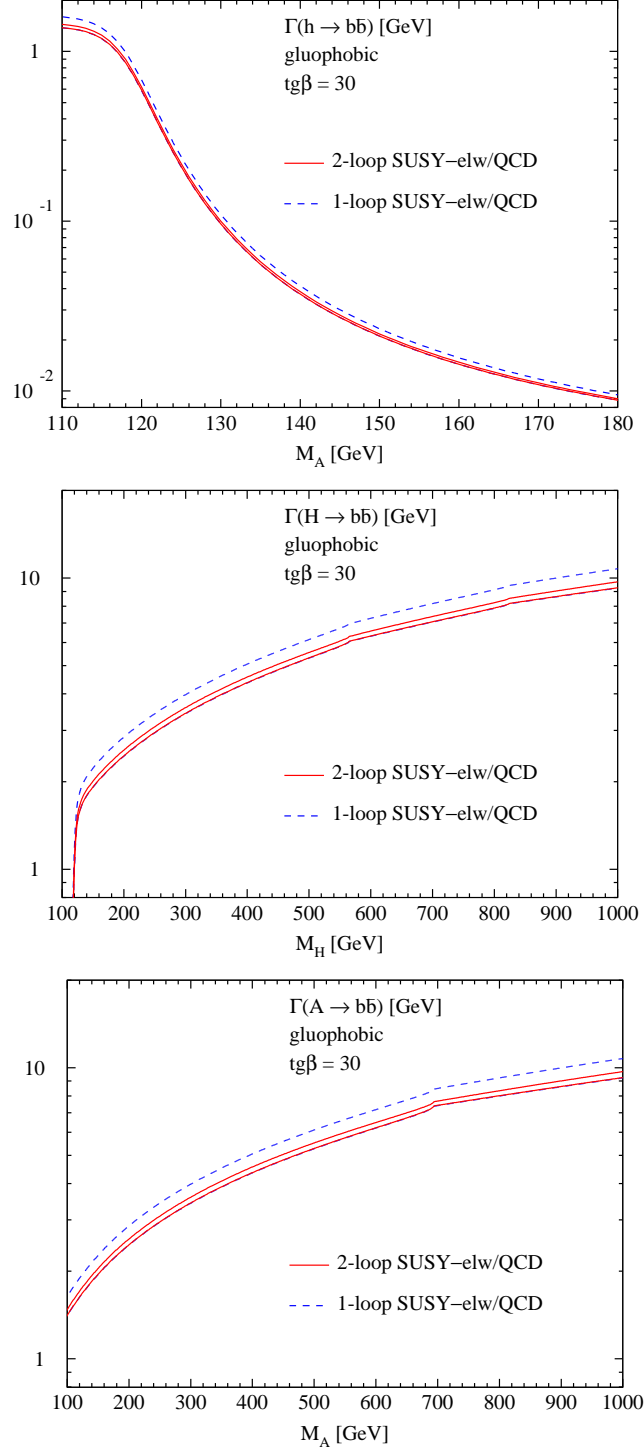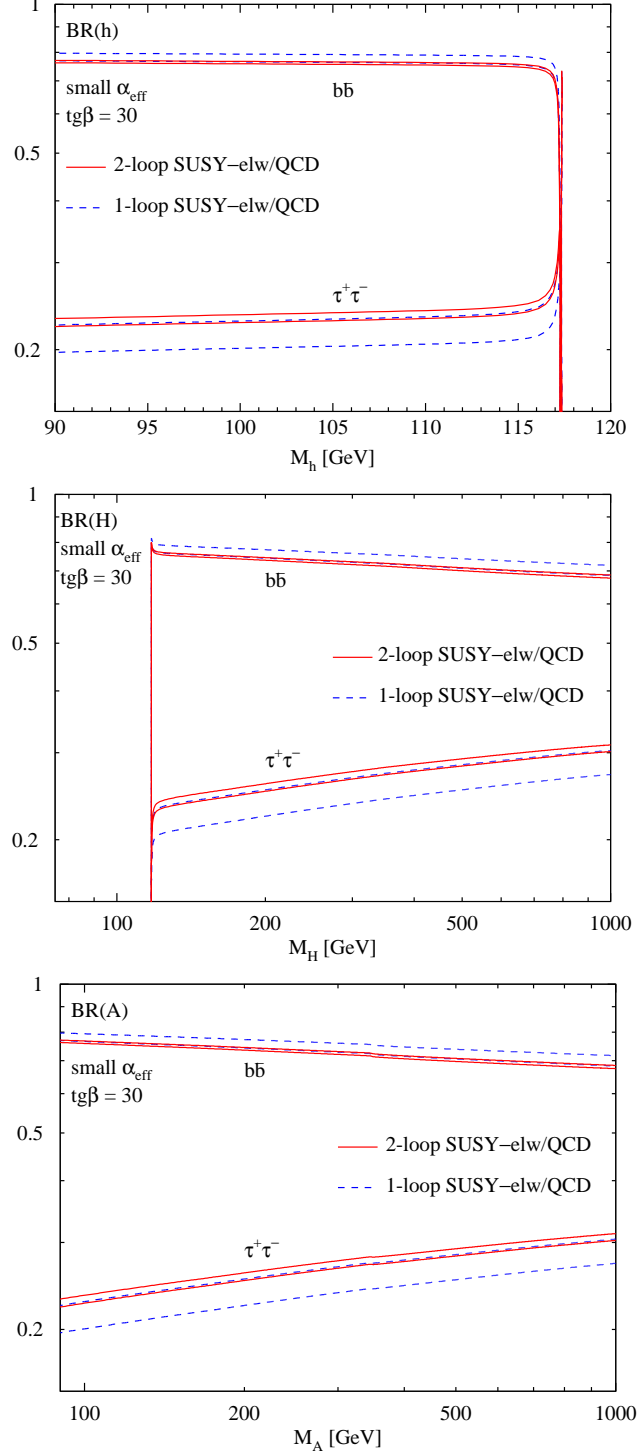
Figure 7: *Partial decay widths of the light scalar h, the heavy scalar H and the pseudoscalar A Higgs bosons to b̄b in the gluophobic scenario. The dashed blue bands indicate the scale dependence at one-loop order and the full red bands at two-loop order by varying the renormalization scales of Δ_b^{QCD} and Δ_b^{elw} independently between 1/2 and 2 times the central scale given by the corresponding average of the SUSY-particle masses.*

Figure 8: *Branching ratios of the light scalar h, the heavy scalar H and the pseudoscalar A Higgs bosons to $b\bar{b}$ and $\tau^+\tau^-$ in the small α_{eff} scenario. The dashed blue bands indicate the scale dependence at one-loop order and the full red bands at two-loop order by varying the renormalization scales of Δ_b^{QCD} and Δ_b^{elw} independently between 1/2 and 2 times the central scale given by the corresponding average of the SUSY-particle masses.*
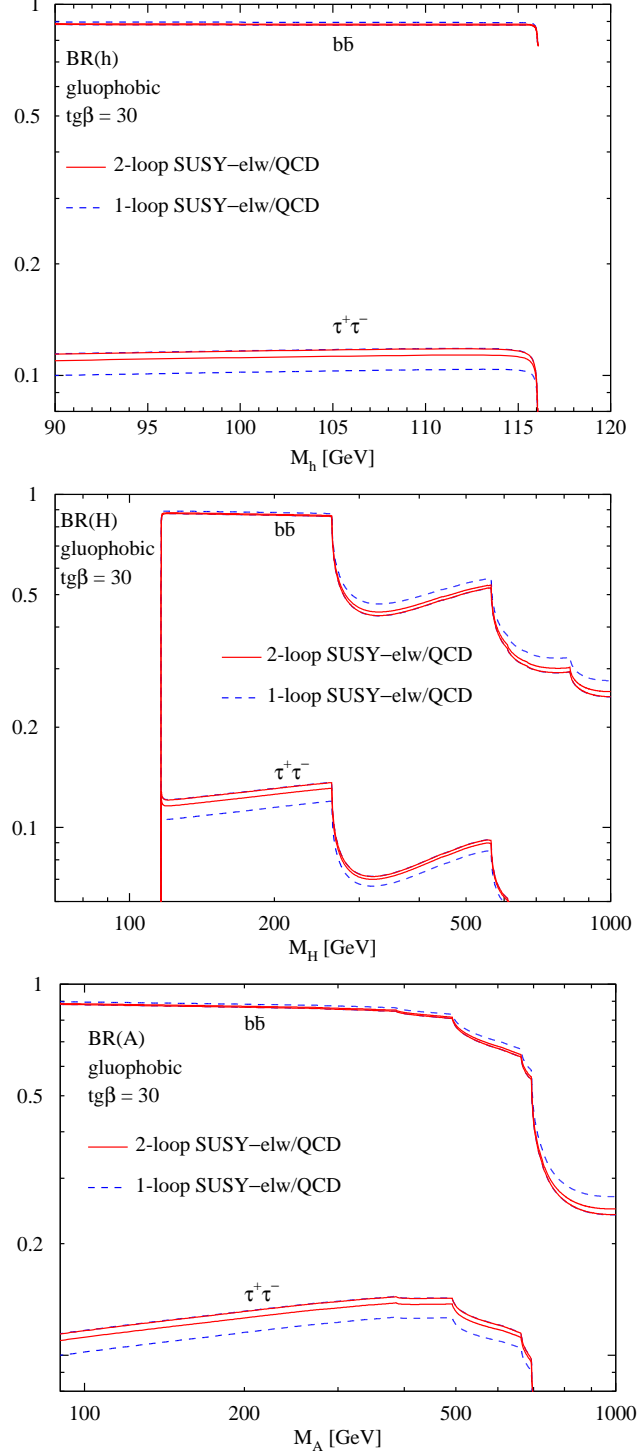
Figure 9: *Branching ratios of the light scalar h, the heavy scalar H and the pseudoscalar A Higgs bosons to $b\bar{b}$ and $\tau^{+}\tau^{-}$ in the gluophobic scenario. The dashed blue bands indicate the scale dependence at one-loop order and the full red bands at two-loop order by varying the renormalization scales of Δ_b^{QCD} and Δ_b^{elw} independently between $1/2$ and 2 times the central scale given by the corresponding average of the SUSY-particle masses.*

way. We have calculated the two-loop SUSY-QCD corrections to these effective bottom Yukawa couplings.

In summary, the significant scale dependence of $\mathcal{O}\left(10\%\right)$ of the NLO predictions for processes involving the bottom quark Yukawa couplings of supersymmetric Higgs bosons require the inclusion of NNLO corrections. For the corrected Yukawa couplings, we find a reduction of the scale dependence to the per-cent level at the NNLO level. The improved NNLO predictions for the bottom Yukawa couplings can thus be taken as a base for experimental analyses at the Tevatron and the LHC as well as the ILC.

Note added

During the publication procedure of our work another paper appeared [32] where the bottom Yukawa couplings have been calculated at two-loop order within SUSY–QCD. After adjusting the different renormalization schemes and approximations we found mutual agreement between the two calculations.

References

[1] P. W. Higgs, Phys. Lett. **12**, 132 (1964), Phys. Rev. Lett. **13**, 508 (1964) and Phys. Rev. **145**, 1156 (1966); F. Englert and R. Brout, Phys. Rev. Lett. **13**, 321 (1964); G. S. Guralnik, C. R. Hagen and T. W. Kibble, Phys. Rev. Lett. **13**, 585 (1964).

[2] Yu.A. Golfand and E.P. Likhtman, JETP Lett. **13** (1971) 323; J. Wess and B. Zumino, Nucl. Phys. B **70** (1974) 39.

[3] See e.g. G. Degrassi, S. Heinemeyer, W. Hollik, P. Slavich and G. Weiglein, Eur. Phys. J. **C28** (2003) 133.

[4] S. P. Martin, Phys. Rev. **D75** (2007) 055005; R. V. Harlander, P. Kant, L. Mihaila and M. Steinhauser, Phys. Rev. Lett. **100** (2008) 191602, Erratum *ibid.* **101** (2008) 039901.

[5] S. Schael *et al.* [ALEPH, DELPHI, L3 and OPAL Collaborations], Eur. Phys. J. **C47** (2006) 547.

[6] M. S. Carena *et al.* [Higgs Working Group Collaboration], arXiv:hep-ph/0010338.

[7] ATLAS Collaboration, Technical Design Report, CERN–LHCC 99–14 (May 1999); G. L. Bayatian *et al.* [CMS Collaboration], J. Phys. **G34** (2007) 995.

[8] G. Aarons *et al.* [ILC Collaboration], arXiv:0709.1893 [hep-ph].

[9] M. Spira, Fortsch. Phys. **46** (1998) 203; A. Djouadi, Phys. Rept. **459** (2008) 1.

[10] J. Guasch, P. Häfliger and M. Spira, Phys. Rev. **D68** (2003) 115001.

[11] M. Carena, D. Garcia, U. Nierste and C. E. Wagner, Nucl. Phys. **B577** (2000) 88.

[12] P. Häfliger and M. Spira, Nucl. Phys. **B719** (2005) 35 and PoS **HEP2005** (2006) 351.

[13] T. Plehn, Phys. Rev. **D67** (2003) 014018; E. L. Berger, T. Han, J. Jiang and T. Plehn, Phys. Rev. **D71** (2005) 115012; S. Dittmaier, M. Krämer, M. Spira and M. Walser, arXiv:0906.2648 [hep-ph].

[14] M. Walser, dissertation ETH Zürich NO. 17592, 2008.

[15] L. J. Hall, R. Rattazzi and U. Sarid, Phys. Rev. **D50** (1994) 7048; R. Hempfling, Phys. Rev. **D49** (1994) 6168; M. Carena, M. Olechowski, S. Pokorski and C. E. Wagner, Nucl. Phys. **B426** (1994) 269.

[16] J. R. Ellis, M. K. Gaillard and D. V. Nanopoulos, Nucl. Phys. **B106** (1976) 292; M. A. Shifman, A. I. Vainshtein, M. B. Voloshin and V. I. Zakharov, Sov. J. Nucl. Phys. **30** (1979) 711 [Yad. Fiz. **30** (1979) 1368]; M. Spira, A. Djouadi, D. Graudenz and P. M. Zerwas, Nucl. Phys. **B453** (1995) 17; B. A. Kniehl and M. Spira, Z. Phys. **C69** (1995) 77; W. Kilian, Z. Phys. **C69** (1995) 89.

[17] A. Bednyakov, A. Onishchenko, V. Velizhanin and O. Veretin, Eur. Phys. J. **C29** (2003) 87; A. Bednyakov and A. Sheplyakov, Phys. Lett. **B604** (2004) 91; A. V. Bednyakov, Int. J. Mod. Phys. **A22** (2007) 5245.

[18] S. P. Martin, Phys. Rev. **D72** (2005) 096008.

[19] A. Bauer, L. Mihaila and J. Salomon, JHEP **0902** (2009) 037.

[20] G. 't Hooft and M.J.G. Veltman, Nucl. Phys. **B153** (1979) 365; G. Passarino and M.J.G. Veltman, Nucl. Phys. **B160** (1979) 151.

[21] A. I. Davydychev and J. B. Tausk, Nucl. Phys. **B397** (1993) 123; F. A. Berends and J. B. Tausk, Nucl. Phys. **B421** (1994) 456.

[22] K. G. Chetyrkin and F. V. Tkachov, Nucl. Phys. **B192** (1981) 159; G. Weiglein, R. Scharf and M. Böhm, Nucl. Phys. **B416** (1994) 606.

[23] D. Noth and M. Spira, Phys. Rev. Lett. **101** (2008) 181801; D. Noth, PhD thesis, Zürich, 2008.

[24] S.P. Martin and M.T. Vaughn, Phys. Lett. **B318** (1993) 331.

[25] A. Djouadi, J. Kalinowski and M. Spira, Comput. Phys. Commun. **108** (1998) 56.

[26] M. Carena, H.E. Haber, S. Heinemeyer, W. Hollik, C.E. Wagner and G. Weiglein, Nucl. Phys. **B580** (2000) 29.

[27] M. Carena, S. Heinemeyer, C.E.M. Wagner and G. Weiglein, Eur. Phys. J. **C26** (2003) 601.

[28] E. Braaten and J.P. Leveille, Phys. Rev. **D22** (1980) 715; N. Sakai, Phys. Rev. **D22** (1980) 2220; T. Inami and T. Kubota, Nucl. Phys. **B179** (1981) 171; S.G. Gorishnii, A.L. Kataev and S.A. Larin, Sov. J. Nucl. Phys. **40** (1984) 329 [Yad. Fiz. **40** (1984) 517]; M. Drees and K.I. Hikasa, Phys. Rev. **D41** (1990) 1547, Phys. Lett. **B240** (1990) 455 [Erratum-ibid. **B262** (1991) 497]; S.G. Gorishnii, A.L. Kataev, S.A. Larin and L.R. Surguladze, Mod. Phys. Lett. **A5** (1990) 2703, Phys. Rev. **D43** (1991) 1633; A.L. Kataev and V.T. Kim, Mod. Phys. Lett. **A9** (1994) 1309; L. R. Surguladze, Phys. Lett. **B341** (1994) 60; K.G. Chetyrkin, J.H. Kühn and A.K. Kwiatkowski, Proceedings of the Workshop "QCD at LEP", Aachen, 1994, arXiv:hep-ph/9407271. K. G. Chetyrkin, Phys. Lett. **B390** (1997) 309; K.G. Chetyrkin and A. Kwiatkowski, Nucl. Phys. **B461** (1996) 3; S.A. Larin, T. van Ritbergen and J.A.M. Vermaseren, Phys. Lett. **B362** (1995) 134.

[29] J. A. Coarasa, R.A. Jimenez and J. Solà, Phys. Lett. **B389** (1996) 312; H. Eberl, K. Hidaka, S. Kraml, W. Majerotto and Y. Yamada, Phys. Rev. **D62** (2000) 055006.

[30] A. Djouadi, J. Kalinowski and P. M. Zerwas, Z. Phys. C **54** (1992) 255; S. Dittmaier, M. Krämer, Y. Liao, M. Spira and P. M. Zerwas, Phys. Lett. B **478** (2000) 247; S. Dawson and L. Reina, Phys. Rev. D **60** (1999) 015003.

[31] S. Dittmaier, M. Krämer and M. Spira, Phys. Rev. D **70** (2004) 074010; S. Dawson, C. B. Jackson, L. Reina and D. Wackeroth, Phys. Rev. D **69** (2004) 074027 and Phys. Rev. Lett. **94** (2005) 031802; J. Campbell, R. K. Ellis, F. Maltoni and S. Willenbrock, Phys. Rev. D **67** (2003) 095002; R. V. Harlander and W. B. Kilgore, Phys. Rev. D **68** (2003) 013001.

[32] L. Mihaila and C. Reisser, JHEP **1008** (2010) 021.